Simplex Example

- iteration-by-iteration standard-form example

Example

$$A = \begin{bmatrix} -2 & 1 & 1 & 0 & 0 \\ -1 & 2 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 1 \end{bmatrix}, \quad b = \begin{bmatrix} 2 \\ 7 \\ 3 \end{bmatrix}, \quad c = \begin{bmatrix} -1 \\ -2 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

iteration 0: $\mathcal{B} = \{ 3, 4, 5 \}, \quad \mathcal{N} = \{ 1, 2 \}$

iteration 1:

- $B = I = B^{-1}$
- current itn: $Bx_B = b \quad \longrightarrow \quad x_B = (2, 7, 3)$
- simplex multipliers: $B^T y = c_B \quad \longrightarrow \quad y = 0$
- reduced costs: $z_N = c_N - N^T y \quad \longrightarrow \quad z_N = (-1, -2)$
- choose $\eta_2 = 2$ to **enter** basis
- search dir: $Bd_B = -a_2 \quad \longrightarrow \quad d_B = (-1, -2, 0)$
- ratio test: $q = \underset{q \mid d_{\beta_q} < 0}{\arg\min} -\frac{x_{\beta_q}}{d_{\beta_q}} \quad \longrightarrow \quad q = 1,\ \beta_q = 3$ **exits** basis
- new basis:$\mathcal{B} = \{ 2, 4, 5 \}, \quad \mathcal{N} = \{ 1, 3 \}$

iteration 2:

- current basis: $\mathcal{B} = \{\, 2, 4, 5 \,\}, \quad \mathcal{N} = \{\, 1, 3 \,\}$
- $B = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad B^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$
- current itn: $Bx_{\scriptscriptstyle B} = b \quad \longrightarrow \quad x_{\scriptscriptstyle B} = (2, 3, 3)$
- simplex multipliers: $B^T y = c_{\scriptscriptstyle B} \quad \longrightarrow \quad y = (-2, 0, 0)$
- reduced costs: $z_{\scriptscriptstyle N} = c_{\scriptscriptstyle N} - N^T y \quad \longrightarrow \quad z_{\scriptscriptstyle N} = (-5, 2)$
- choose $\eta_1 = 1$ to **enter** basis
- search dir: $Bd_{\scriptscriptstyle B} = -a_1 \quad \longrightarrow \quad d_{\scriptscriptstyle B} = (2, -3, 1)$
- ratio test: $q = \underset{q \in \{\, 1, \ldots, m \,\} | d_{\beta_q} < 0}{\arg\min} -\frac{x_{\beta_q}}{d_{\beta_q}} \quad \longrightarrow \quad q = 2,\ \beta_q = 4$ **exits** basis
- new basis: $\mathcal{B} = \{\, 2, 1, 5 \,\}, \quad \mathcal{N} = \{\, 4, 3 \,\}$

iteration 3:

- current basis: $\mathcal{B} = \{\, 2, 1, 5 \,\}, \quad \mathcal{N} = \{\, 4, 3 \,\}$
- $B = \begin{bmatrix} 1 & -2 & 0 \\ 2 & -1 & 0 \\ 0 & 1 & 1 \end{bmatrix}, \quad B^{-1} = (1/3) \begin{bmatrix} -1 & 2 & 0 \\ -2 & 1 & 0 \\ 2 & -1 & 3 \end{bmatrix}$
- current itn: $Bx_B = b \quad \longrightarrow \quad x_B = (4, 1, 2)$
- simplex multipliers: $B^T y = c_B \quad \longrightarrow \quad y = (4/3, -5/3, 0)$
- reduced costs: $z_N = c_N - N^T y \quad \longrightarrow \quad z_N = (5/3, -4/3)$
- choose $\eta_2 = 3$ to **enter** basis
- search dir: $Bd_B = -a_3 \quad \longrightarrow \quad d_B = (1/3, 2/3, -2/3)$
- ratio test: only candidate basic variable is $q = 3$, $\beta_q = 5$ **exits** basis
- new basis: $\mathcal{B} = \{\, 2, 1, 3 \,\}, \quad \mathcal{N} = \{\, 4, 5 \,\}$

iteration 4:

- current basis: $\mathcal{B} = \{\,2, 1, 3\,\}$, $\quad \mathcal{N} = \{\,4, 5\,\}$
- $B = \begin{bmatrix} 1 & -2 & 1 \\ 2 & -1 & 0 \\ 0 & 1 & 0 \end{bmatrix}$, $\quad B^{-1} = \begin{bmatrix} 0 & 1/2 & 1/2 \\ 0 & 0 & 1 \\ 1 & -1/2 & 3/2 \end{bmatrix}$
- current itn: $Bx_B = b \quad \longrightarrow \quad x_B = (5, 3, 3)$
- simplex multipliers: $B^T y = c_B \quad \longrightarrow \quad y = (0, -1, -2)$
- reduced costs: $z_N = c_N - N^T y \quad \longrightarrow \quad z_N = (1, 2)$
- $z_N \geq 0 \quad \longrightarrow \quad$ basis is **optimal**